figure in the 4.0 to 6.0-GHz frequency range with a maximum flat gain. The analytic approach used in [2] was the basis of the input network design. This input network was designed to match the conjugate of the optimum source impedance for low noise at 5.0 GHz. Interpolation was used to derive this impedance from the published data at 4.0 and 6.0 GHz. The network element values were derived from an $n=3$, 0.01-dB ripple, Chebyshev low-pass prototype [8]. The first impedance inverter (K_{34}) was omitted from the final design because its 50.75 Ω impedance contributed little to the overall match.

The transistor output equivalent circuit was derived from the conjugate of the reflection coefficient for a matched output with minimum noise. The unilateral gain approximation as described in [9] was used to find the maximum amplifier gain at 4.0 and 6.0 GHz. The source reflection coefficient derived from the conjugate of the transistor input equivalent circuit at 4.0 and 6.0 GHz was used in this unilateral gain approximation. This required 3.55 dB of loss at 4.0 GHz. A maximum gain of 8.45 dB at 6.0 GHz was expected for the amplifier. The diplexer network needed to provide 2.85 dB of the 3.55-dB loss, and so a normalized bandwidth of 0.864 was used. Although the cascade of two identical single-stage amplifiers provided acceptable results without adjustment of any of the circuit elements, optimization improved the gain flatness and output reflection coefficient. The noise figures of 2.1 dB and 4.0 GHz and 2.35 dB at 6.0 GHz were changed to 1.9 dB at 4.0 GHz and 2.48 dB at 6.0 GHz during optimization. The relatively high VSWR of this amplifier's input, as shown in Fig. 7(b), is due to providing an optimum source impedance for noise minimization.

The formulas presented in this paper are useful whenever the equivalent circuit of the device is a parallel RC network. Therefore, bipolar transistor amplifiers operating well below the device's f_T may contain the circuit just presented as an input matching network.

IV. Conclusion

A simple output network was shown to simultaneously provide gain-compensation and a predictable amplifier design. The feasibility of this design method was demonstrated by cascading two identical single-stage amplifiers and calculating the total amplifier S-parameters before and after optimization. Although the input and output networks were designed by treating the amplifier as if these networks did not interact, the actual results agreed well with simple theory. Explicit formulas for the design of lumped and distributed output networks were presented.

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Application of the Galerkin Method for Determination of Quasi TE_{i0k} Mode Frequencies of a Rectangular Cavity Containing a Dielectric Sample

ANDRZEJ KEDZIOR AND JERZY KRUPKA

Abstract —**A new method determination of quasi TE_{i0k} mode frequencies of a rectangular cavity containing a dielectric sample is presented. A centrally loaded dielectric sample fills completely only one dimension of a cross section of the cavity. The calculations are based on the Galerkin method using a new suitable set of basis functions. The theoretical results are illustrated by experiments. The obtained results of calculations and experiments demonstrate the advantages of the new basis as compared with the classical one. The presented method may be applied for the analysis of two-dimensional boundary problems for various resonant cavities with inhomogeneous filling.**

I. Introduction

It is often necessary to determine the permittivity of rectangular samples of precisely defined dimensions at microwave frequencies. The parameters of such samples are determined most frequently by resonance method. The form of the sample determines the rectangular form of the cavity [2], [4], [5]. An accurate determination of the permittivity of precisely defined dimension samples is often difficult, especially at the low microwave frequencies. At these frequencies cavity dimensions are usually larger than respective sample dimensions. This leads to the necessity of using approximate methods for determining the resonant frequency of the cavity in relation to the permittivity. Since it is easier to determine the resonant frequency of the cavity at a fixed permittivity, the paper presents the solution of such problems.

II. Theory

In this section is presented a method of determining the angular frequencies of quasi TE_{i0k} modes ($i, k =$ odd numbers) of rectangular cavity with infinity conducting walls, filled with a dielectric in the way shown in Fig 1(a). This problem is reduced to determination of the eigenvalues of the boundary problem

$$L\phi = j\omega M\phi$$
$$\boldsymbol{n} \times \boldsymbol{E} = 0, \qquad \text{on } S \tag{1}$$

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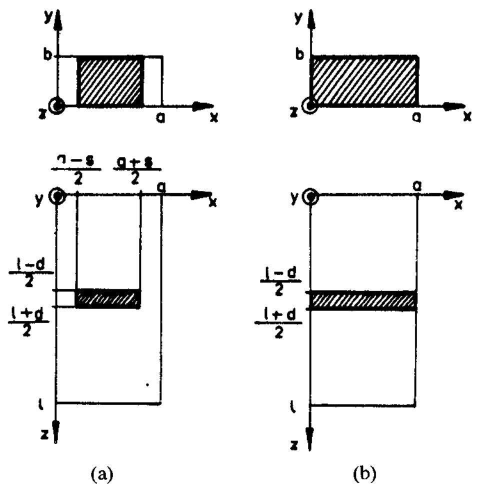

Fig. 1. Rectangular resonant cavity with a centrally located dielectric sample. (a) The problem analyzed in this paper. (b) Basic problem with analytical solutions.

where

$$L=\begin{bmatrix} 0 & \nabla x \\ \nabla x & 0 \end{bmatrix} \quad M=\begin{bmatrix} \epsilon_0\epsilon & 0 \\ 0 & -\mu_0 \end{bmatrix} \quad \phi=\begin{bmatrix} E \\ H \end{bmatrix}$$

$$\epsilon=\begin{cases} \epsilon_r, & \text{for } \frac{a-s}{2}\leqslant x \leqslant \frac{a+s}{2} \text{ and } \frac{l-d}{2}\leqslant Z \leqslant \frac{l+d}{2} \\ 1, & \text{in the remaining part of the cavity} \end{cases}$$

where

E, H electric and magnetic fields inside the cavity.
S surface of the cavity.

The solution of the above problem has been obtained by the Galerkin method. Following this method, the function ϕ is expanded into the finite sum of the functions

$$\phi=\sum_{n=1}^{N}\alpha_n\phi_n \tag{2}$$

where

$\{\alpha_n\}$ set of coefficients to be determined.
$\{\phi_n\}$ set of basis functions. It is a set of functions belonging to the field of definition of the operator L and satisfying the proper boundary conditions on S.

Currently the solutions of the boundary problem (1) for an empty cavity have been used to analyse similar problems [1],[2]. In the approach presented in this paper a new basis is formed by the functions being the solutions of the boundary problem (1) for the cavity with a transverse cross section completely filled by a dielectric as shown in Fig 1(b). Since we are looking for the frequencies of specific types of modes, the basis can be reduced to the class of rotational functions corresponding to electric and magnetic fields of quasi TE_{i0k} modes ($i, k =$ odd numbers) of the resonator from Fig 1(b). Functions ϕ may be thus written in the form [1]

$$\phi=\sum_{n=1}^{N}\left[\alpha_n^E,\alpha_n^H\right]\begin{bmatrix} E_n \\ H_n \end{bmatrix}. \tag{3}$$

Substituting (3) into (1) and forming inner products in the cavity volume V one obtains a system of linear equations

$$\sum_{n=1}^{N}\left(A_{mn}-\delta_{mn}\frac{1}{\omega^2}\right)\alpha_n^H=0, \qquad m=1,2,\cdots,N \tag{4}$$

where

$$A_{mn}=\frac{\langle \epsilon\epsilon_0\phi_n,\phi_m\rangle}{\omega_m\omega_n}, \qquad \phi_n=[E_n,H_n], \qquad \phi_m=\begin{bmatrix} E_m \\ H_m \end{bmatrix}$$

where

ω_m, ω_n angular frequencies corresponding to the individual types of quasi TE_{i0k} modes of the cavity from Fig 1(b).
$\langle\ \rangle$ inner product of functions $\langle f,g\rangle \underset{df}{=} \int_V fg^* dv$.

The systems of equations (4) have nontrivial solutions for ω values if

$$\det\left|A_{mn}-\delta_{mn}\frac{1}{\omega^2}\right|=0 \tag{5}$$

so the set of $1/\omega^2$ values being eigenvalues of matrices A_{mn} is the solution of problem (1).

III. Choice of Basis Functions

The most essential problem appearing at the application of the Galerkin method consists in the choice of N basis functions which provide the best accuracy for the calculation of angular frequencies. The relatively easiest situation occurs in the case of calculating the lowest frequency value corresponding to quasi TE_{101} mode. In this case it is known that the frequency value calculated by the Galerkin method is in excess of the accurate value [2]. As a consequence, there appears a possibility of simple evaluation of various bases. The basis which provides the lower value of frequency is better. In the case of calculating the higher mode frequencies it is impossible to establish if the calculated value of the frequency is in excess of the accurate value. Therefore, in the case of higher order modes, the evaluation of the convergence of the solutions for various sets of basis functions has been verified experimentally.

Fig. 2 presents the values of calculation errors of angular frequency shift of a quasi TE_{101} cavity with a sample of the width $s/a=0.4$, using various sets of basis functions with both classical (empty cavity modes) and new bases. The "exact" value of angular frequency ω_e, used as a reference for error calculations, is assumed to equal the lowest angular frequency value among the values of corresponding various bases. The maximum number of basis functions N was limited to 15, and three various configura-

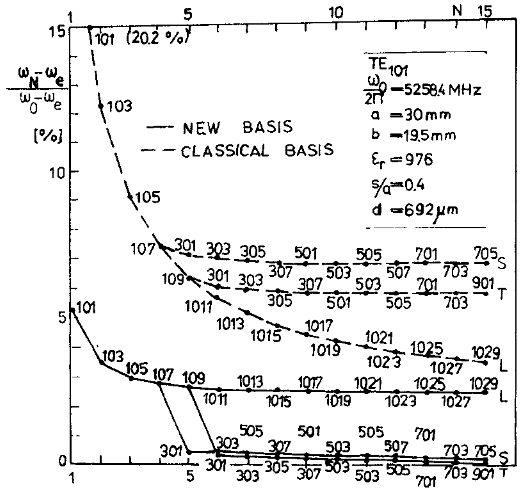

Fig. 2. Errors of frequency shift determination in quasi-TE_{101} cavity containing a sample width $s/a = 0.4$ for various sets of basis functions. The results are refered to the "exact" value of angular frequency shift. Numbers on the curves denote subscripts $i0k$ of TE_{i0k} basis functions (classical basis) and quasi-TE_{i0k} basis function (new basis).

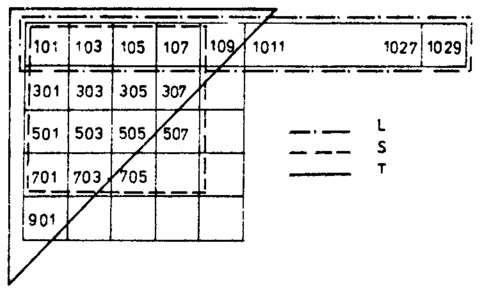

Fig. 3. Various sets of basis functions consisting of TE_{i0k} and quasi-TE_{i0k} modes.

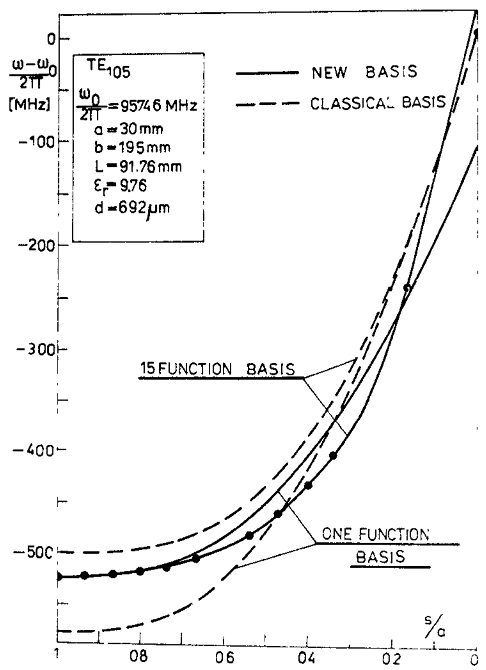

Fig. 4. Frequency shift of quasi-TE_{105} cavity calculated using various bases. a) The best new 15-function basis. b) The best classical 15-function basis. c) A new single-function basis. d) The classical single-function basis. The point denotes the experimental results.

tions of sets for classical and new bases denoted by letters L, T, S were used (Fig. 3). For each of the three configurations the number of basis functions was subsequently reduced to one in a sequence marked in Fig. 2. The three and four figure numbers on Fig. 2 and Fig. 3 denote subscripts $i0k$ of TE_{i0k} basis functions (classical basis—empty cavity modes) and quasi-TE_{i0k} basis functions (new basis).

It follows from Fig. 2 that the use of a new basis yields best results in the case of T configuration, whereas the classical basis yields the best results in the case of L configuration. The classical 15-function basis L provides the same calculations accuracy as the new basis composed of only two functions.

Fig. 4 presents the values of the frequency shift of a quasi TE_{105} cavity with samples of various width s, calculated for the best two 15-function bases among those compared in Fig. 2 (new and classical ones) and two single function bases. The experimental data are marked by points. The results of calculations and experiments confirm the results presented in Fig. 2.

IV. Conclusions

The performed calculations and measurements lead to the conclusion that the modification of the basis in the Galerkin method, presented in this paper, yields much more accurate results of calculations than the use of the classical basis (with some restrictions imposed on sample dimensions, mentioned in the paper). The presented method may find applications in the analysis of two-dimensional boundary problems for cavities with regular inhomogeneous filling. Such problems are typically encountered in the measurements of the permittivity of dielectrics.

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The Conservation of Complex Power Technique and *E*-Plane Step-Diaphragm Junction Discontinuities

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***Abstract*—The singular integral equation solution due to L. Lewin and his colleagues for the *E*-plane step-diaphragm junction discontinuity are extended by the conservation of complex power technique (CCPT). The singular integral equation method provides formulas for the junction susceptance (both with and without a diaphragm) which are valid only in the**

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